# Two Tape TM Exercise
Jay Bagga

1 Two Tape TM

Let us build a two tape TM for the following language L over the alphabet $\Sigma = \{a, b, c\}$.

$$L = \{wcw | w \in \{a, b\}^*\}.$$

You will create this TM with JFLAP. First think about an informal algorithm that uses two tapes and accepts such words. Essentially, you have to check that there is exactly one c and the string to the left of c is the same as the string to the right of c and that this string belongs to $\{a, b\}^*$. We assume that the input is on Tape 1 and that Tape 2 is blank initially. Now we can use the 2nd tape to make a copy of the string of symbols on the first tape that appear before the c. Then we match the string to the right of c on the first tape with the string on the 2nd tape.

Let us proceed in steps as follows.

1. In JFLAP start by choosing to create a two tape TM. Create an initial state $q0$. What are the possibilities for the first symbol in the input? You can distinguish three cases: The symbols is a c, or it is an a or b, or it is a blank. In the first case, the input must be exactly c to be accepted. So create a new state $q1$ where you can read the c and move right on Tape 1, and stay on Tape 2. In the second case, you want to copy the a or b on Tape 2 and move right on both tapes. So add a new state $q2$ with appropriate transitions. What do we do in the third case? Once you are done with this step, compare your TM with that shown below.
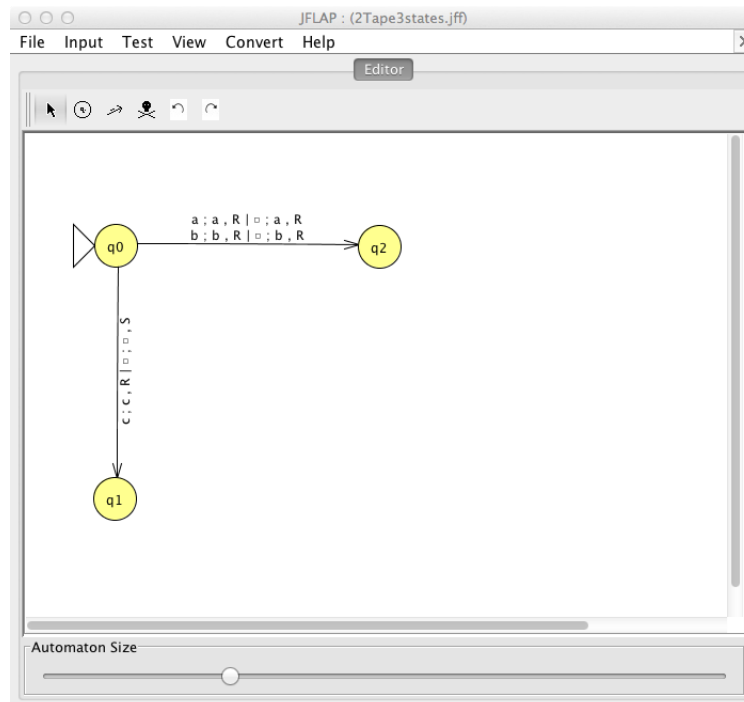
2. What is the next step? For $q1$, what should you expect? The 1st tape must have a blank to accept. Introduce a new accept state $q3$ and add a transition appropriately.

3. For $q2$, you must keep reading and copying a's or b's until you come to a c, when you move to a new state $q4$. Add those transitions to $q2$. Compare your construction so far to the one shown below. Explain the meaning of the transition from $q2$ to $q4$.
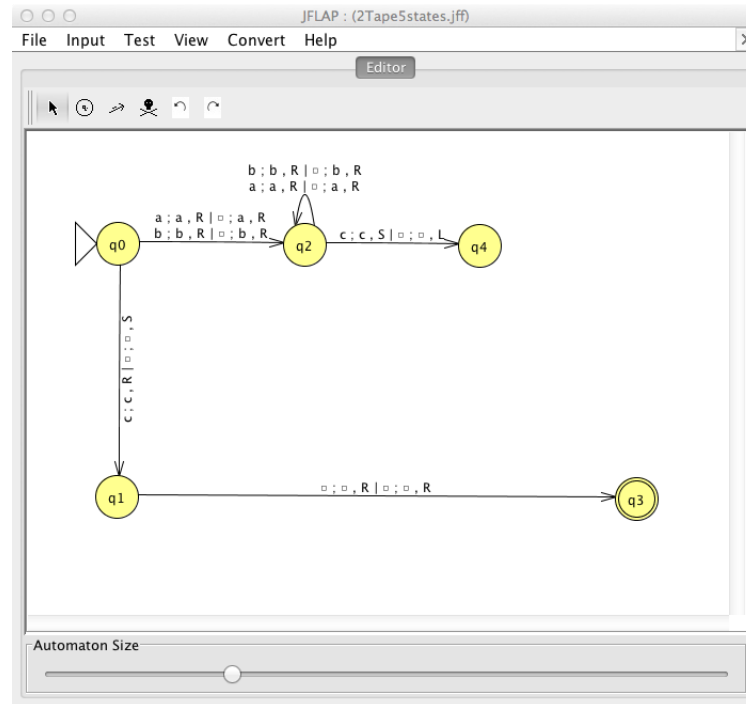
4. At this stage, we want to stay on c in the first tape, but move all the way back to the beginning of the second tape, by looking for a blank. When that happens, we move to a new state where we compare the string to the right of c on Tape 1 to the string on Tape 2. Finally we accept if the two match. Compare your result so far with that shown below.
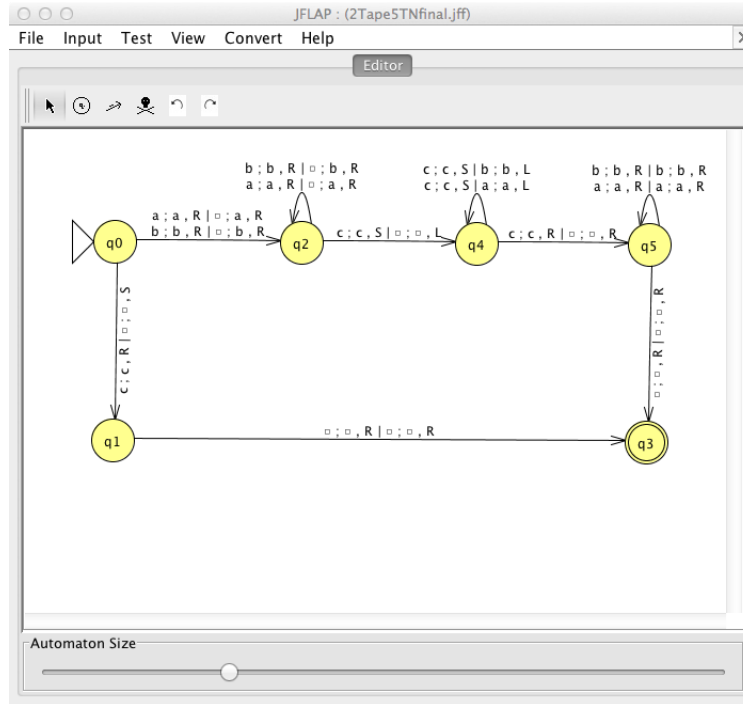
5. Test your two tape TM with multiple inputs and compare your results to those shown below.

6. Test some more inputs.

2 Additional Problem

1. Repeat the above steps for the language

$$L = \{wccw^R | w \in \{a, b\}^*\},$$

where w^R is the word w written in reverse.

3 References

1. Introduction to the Theory of Computation (Third Edition), Michael Sipser. Cengage Learning. 2013.

2. JFLAP - An Interactive Formal Languages and Automata Package, Susan H. Rodger and Thomas W Finley. Jones and Bartlett Publishers. 2006